Understanding Information Disclosure from Secure Computation Output

Analytical and Data-Driven Analysis

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Motivation

Information disclosure analysis

Framework for quantifying disclosure Case study: average salary computation Advanced statistical functions

Conclusions and future work

(Secure) multi-party computation, in a nutshell





$$(s_1, s_2, s_3, s_4, s_5)$$



Multi-party computation (MPC)

Multiple participants **jointly** evaluating an **arbitrary** function on private inputs while revealing only the output(s).

- FHE, garbled circuits, secret sharing
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What do we *really* mean by "secure"?



 No information disclosed throughout computation, other than the output

What do we *really* mean by "secure"?



- No information disclosed throughout computation, other than the output
- But does the **output itself** contain sensitive information?
- Can we **quantify** this disclosure in a meaningful way?

- Develop an information-theoretic approach to measure disclosure
- Apply technique to a practically significant function (the average)
- Extend analysis to complex statistical functions
- Determine and apply appropriate mitigation strategies

Function *f* evaluated on private data

[Bac24, Part II]

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[Bac24, Part II]





- How do we **distinguish** participants from each other?
- Partition parties P into:



S₂





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 - spectators S

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Entropy!



C. Shannon. Photo: Alfred Eisenstaedt, 1963



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Entropy!

Shannon (discrete):

$$H(X) = -\sum_{x \in \mathcal{X}} \Pr(X = x) \log \Pr(X = x)$$

Differential (continuous):
$$h(X) = -\int f(x) \log f(x) dx$$

 J_{X}



C. Shannon. Photo: Alfred Eisenstaedt, 1963

Putting everything together

- Attackers X_A , targets X_T , and spectators X_S
- Treat the **output** as a random variable: $f(\mathbf{X}_A, \mathbf{X}_T, \mathbf{X}_S) = O$

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Attackers' weighted average entropy

[AH17]

 $H(\mathbf{X}_{\mathcal{T}} \mid \mathbf{X}_{\mathcal{A}} = \mathbf{x}_{\mathcal{A}}, O) \implies \overset{\text{"how much information } \mathcal{A} \text{ learns}}{\text{about the target, given } \mathbf{x}_{\mathcal{A}} \text{ and } O"$

Absolute entropy loss

[BBZ24a; BBZ24b]

 $H(\mathbf{X}_{\mathcal{T}}) - H(\mathbf{X}_{\mathcal{T}} \mid \mathbf{X}_{\mathcal{A}} = \mathbf{x}_{\mathcal{A}}, O) \implies \text{``the total amount of information disclosed about the target, given <math>\mathbf{x}_{\mathcal{A}}$ and O''

Absolute entropy loss \iff mutual information between X_T and O (conditioned on $X_A = x_A$)

Case study: the average salary computation

- 2016 Boston gender pay gap survey
- Analyzed the private wages based on gender and race using MPC
- Average salary computation

Mayor Walsh & Boston Women's Workforce Council Release 2016 Gender Wage Gap Report; New Partnership with BU

Thursday, January 5, 2017 – Mayor Martin J. Walsh and the Boston Women's Workforce Council (BWWC) released the 2016 gender wage report and announced a new academic partnership with Boston University, where the BWWC will now be hosted within the BU Hariri Institute for Computing.

Source: Boston University, 2017

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$$O = \sum_{i} X_{T_i} + \sum_{j} X_{A_j} + \sum_{k} X_{S_k}$$

$$H(\mathbf{X}_{T} \mid \mathbf{X}_{A} = \mathbf{x}_{A}, O) = H(\mathbf{X}_{T} \mid \sum_{i} X_{T_{i}} + \sum_{k} X_{S_{k}})$$

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- Intuition: an adversary can "remove" their influence
- May not always be the case (depending on f)



- Model inputs by common distributions:
 - Poisson
 - Uniform
 - Gaussian
 - Log-normal [Cao+22]

- Model inputs by common distributions:
 - Poisson
 - Uniform
 - ≻→− Gaussian
 - Log-normal [Cao+22]
- For a single evaluation, information disclosure is **independent** of
 - the distribution parameters
 - the distribution itself
- Disclosure is proportional to the number of **spectators**



Figure 1: Absolute entropy loss (lower is better)

"What about differential privacy?"



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- Useful for large databases (e.g., $n \ge 10,000)$. . .
- ... but **destroys** the utility of the result for small n (up to 100% error!)
- Our goal: first determine if a function discloses too much information
- We have an effective means of lowering disclosure for the average (increasing participants)

- First wage gap study was successful
- Conducted again the following year with an extended set of participants

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- Combination of new and old parties participating in the computation
- Spectators present in the first, second, and both evaluation(s)

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An interesting question



An interesting question



"What happens if everyone else participates again, but without me?"

 $f(s_2, s_3, s_4, s_5) = o'$



Target participates in one or both evaluations

- Vary the **ratio** of shared spectators $S_{1,2}$ to the (fixed) total number of spectators

Target's initial entropy
 After first evaluation
 Participating in both comps.
 Participating second comp. only
 Participating first comp. only



Figure 2: Conditional entropies, 6 total spectators

Target participates in one or both evaluations

- Vary the **ratio** of shared spectators $S_{1,2}$ to the (fixed) total number of spectators
- Largest protection at 50% overlap
- Undesirable disclosure at extrema
- Target's initial entropy
 After first evaluation
 Participating in both comps.
 - Participating second comp. only
 - Participating first comp. only



Figure 2: Conditional entropies, 6 total spectators

What are some logical successors to the average?

Next step: advanced statistical measures

What are some logical successors to the average?

- Order statistics (max/min, median)

$$f_{\max}(\mathbf{x}) = \max_i x_i$$

- Variability measures (variance)

$$f_{\sigma^2}(\mathbf{x}) = \frac{1}{n} \sum_{i} (x_i - f_{\mu}(\mathbf{x}))^2$$

- Multidimensional functions

$$f_{(\mu,\sigma^2)}(\mathbf{x}) = (f_{\mu}(\mathbf{x}), f_{\sigma^2}(\mathbf{x}))$$

Data-driven estimators of entropy



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Problem

Function could produce discrete outputs from continuous inputs, producing a "**mixture**"







Computer Science > Information Theory

[Submitted on 19 Sep 2017 (v1), last revised 9 Oct 2018 (this version, v3)]

Estimating Mutual Information for Discrete-Continuous Mixtures

Weihao Gao, Sreeram Kannan, Sewoong Oh, Pramod Viswanath

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Function could produce discrete outputs from continuous inputs, producing a "**mixture**"





Continuous k-NN [GOV18]

Computer Science > Information Theory

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But recall (from slide 8)...

mutual information ⇔ absolute loss

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An adversary **maximizes** the information learned by **minimizing** their influence.*

^{*}Inverse behavior for the minimum

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In fact, the information A learns is
 bounded by observing the output,
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Figure 3: Uniform $\mathcal{U}(0,7)$, $H(X_T | X_A = x_A, O)$

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Figure 3: Normal $\mathcal{N}(0, 4.0)$, $H(\mathbf{X}_{\mathcal{T}} \mid \mathbf{X}_{\mathcal{A}} = \mathbf{x}_{\mathcal{A}}, O)$

Surprising observations: simultaneous release

Variance and mean release

The total disclosure from **individual** function outputs f_{μ} and f_{σ^2} is **at least** the amount of information disclosed from a **joint release** $f_{(\mu,\sigma^2)}$?



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- The quantity $H_{f_{\mu}} + H_{f_{\sigma^2}}$ itself isn't practically significant



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 A can learn more information
 about the target

$$\begin{array}{|c|c|c|c|c|} & & & & & & \\ \hline \bullet & & \\ \bullet & & \\ \hline \bullet & \\ \hline \bullet & \hline \bullet & \hline$$



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$$\begin{array}{c|c} \bullet & H_{f_{\mu}} + H_{f_{\sigma^2}} \\ \hline \bullet & H_{f_{(\mu,\sigma^2)}} \end{array} \qquad \begin{array}{c|c} \bullet & |S| = 2 \\ \hline \bullet & |S| = 5 \end{array}$$



More information is revealed from the **joint release** $f_{(\mu,\sigma^2)}$ than from the **individual** function outputs f_{μ} and f_{σ^2} .

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- The quantity $H_{f_{\mu}} + H_{f_{\sigma^2}}$ itself isn't practically significant
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- Does there exist some nontrivial function(s) f? that leaks the target's information entirely?



- Theoretical framework from our comprehensive analysis of the average
- Much to learn for complex functions

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Further analysis of complex functions

- Derive analytical expressions the entropy
- Estimators suffer from the "curse of dimensionality"
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Mitigation strategies

- Adding noise (DP)
- Synthetic inputs
- Modifying the function

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Alternate metrics

- (min-, g-, cross) entropies

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Other research interests

- MPC techniques and applications [Bac24, Part I], [BBY23]
- MPC compiler development
- Threshold FHE

Multi-Party Replicated Secret Sharing over a Ring with Applications to Privacy-Preserving Machine Learning

Authors: Alessandro Baccarini (University at Buffalo (SUNY)), Marina Blanton (University at Buffalo (SUNY)), Chen Yuan (University at Buffalo (SUNY))

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🙀 applied-crypto-lab

picco

This repository corresponds to the PICCO compiler for secure multi-party computation published in 2013 with more recent efficiency improvements.

☆ 11 stars 양 4 forks

https://github.com/applied-crypto-lab/picco

Thank you!

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